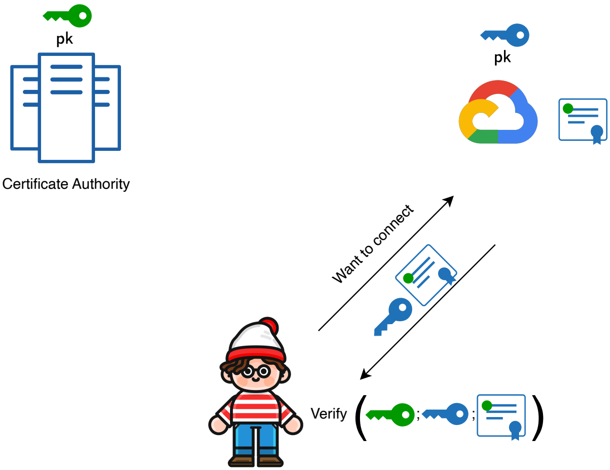
RSA-Based Dynamic Accumulator without Hashing into Primes

Victor Youdom Kemmoe   Anna Lysyanskaya

Brown University

ACM CCS 2024

Motivation (Case of WebPKI certificate revocation)



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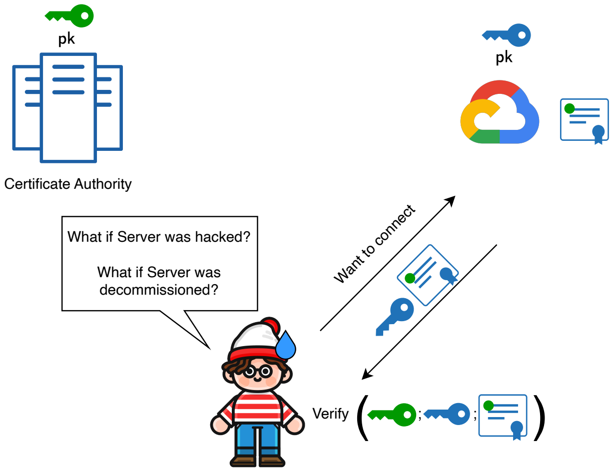

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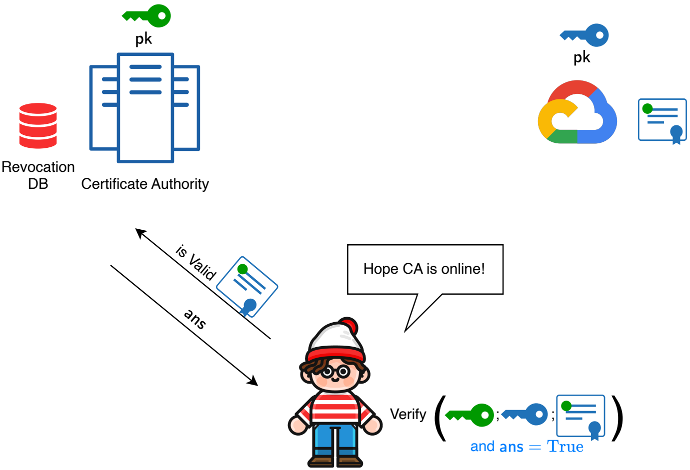

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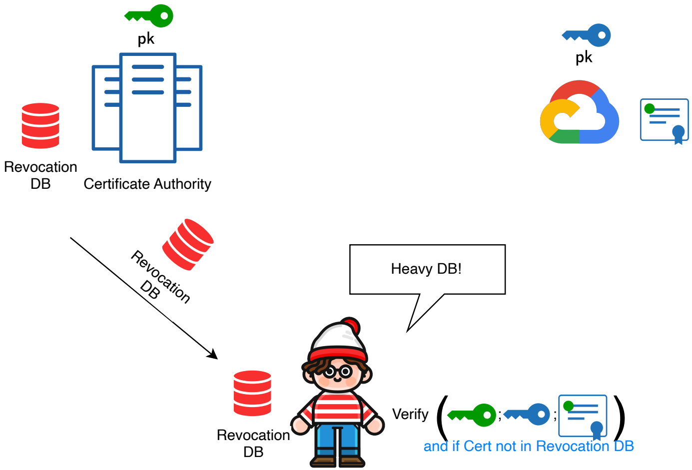

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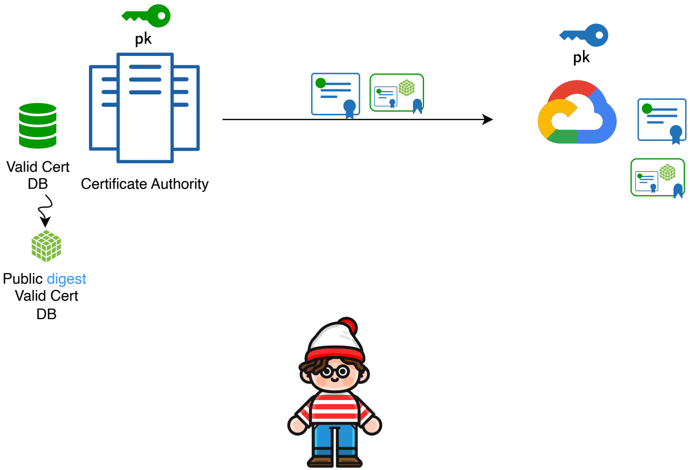

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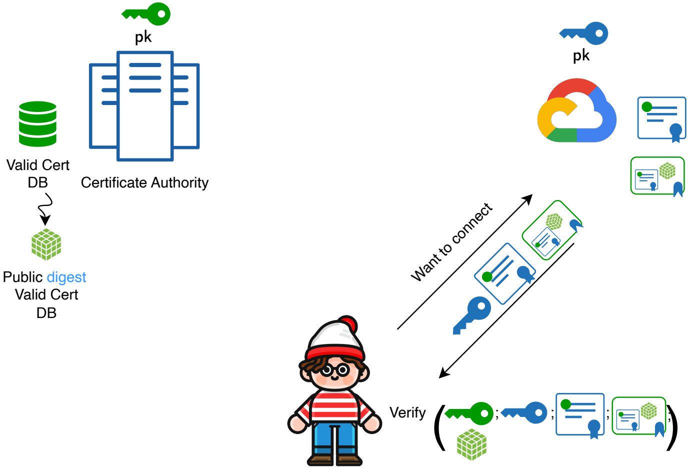

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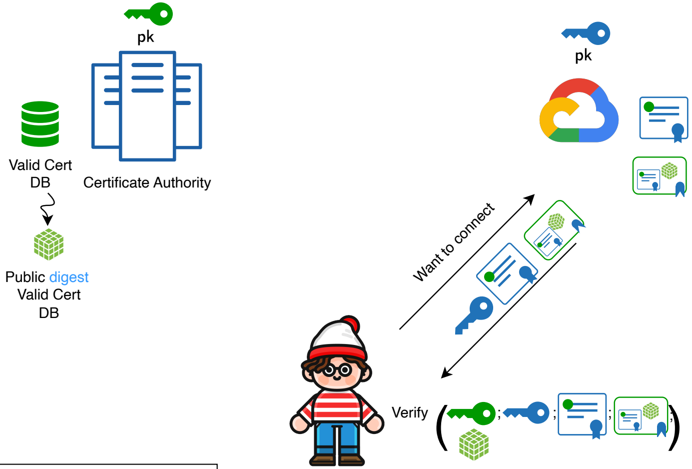
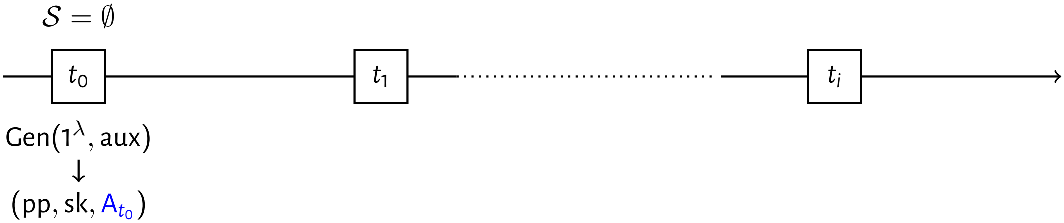


CA is not required to be always online

Digest has a small (constant) size

Dynamic Accumulator

Syntax [DHS15, BCD+17, BKR23]



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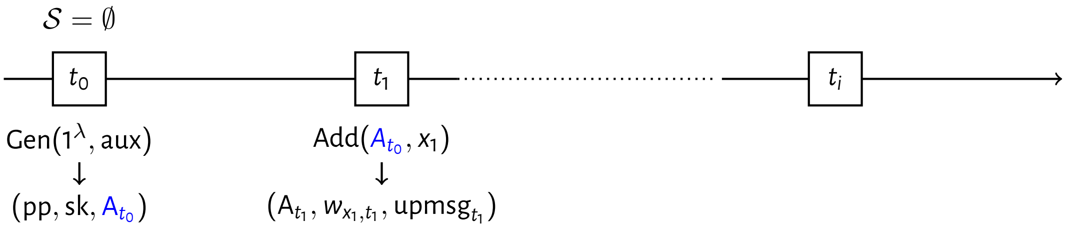

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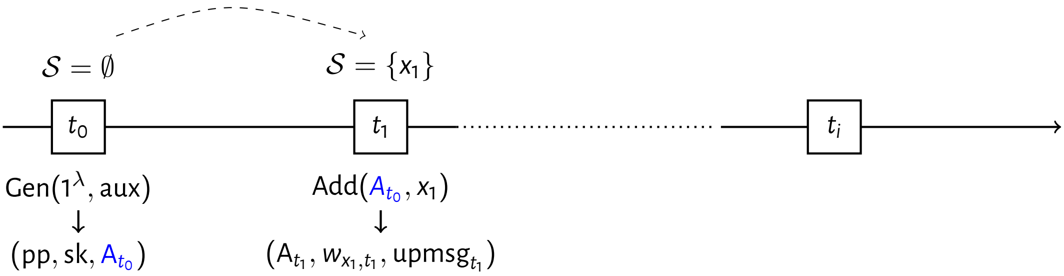

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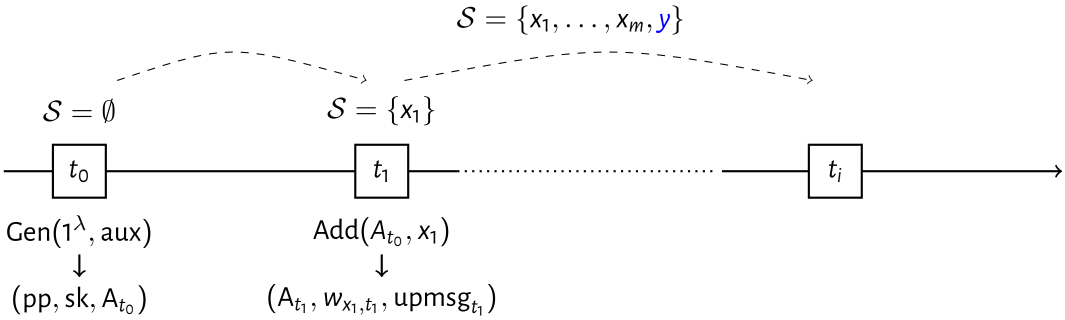

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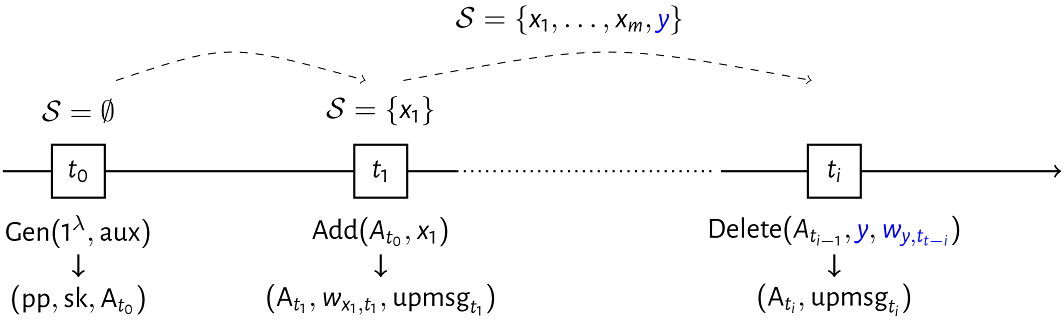

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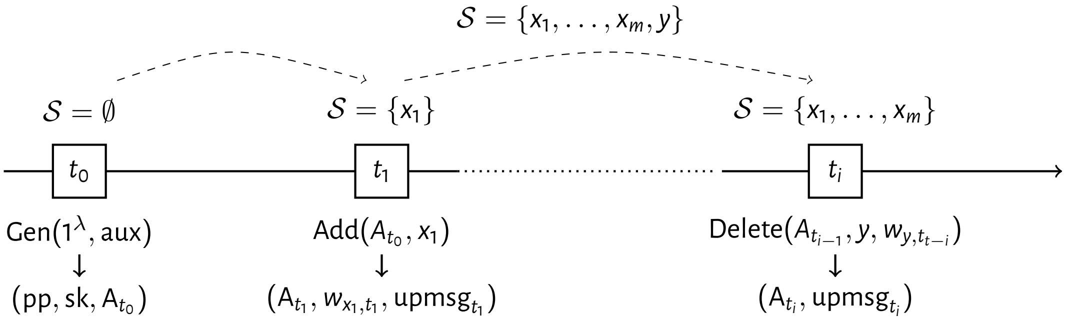

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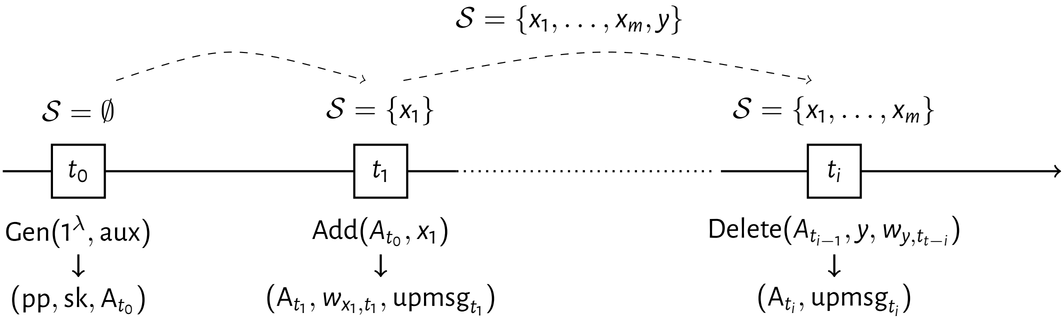

- $\mathsf{MemWitUp}(x, w_{x,t}, \mathsf{upmsg}_{t+1}) \rightarrow w_{x,t+1}$
- $\mathsf{MemVerify}(\mathsf{A}_t, x, w_{x,t}) \rightarrow \mathsf{Accept}/\mathsf{Reject}$

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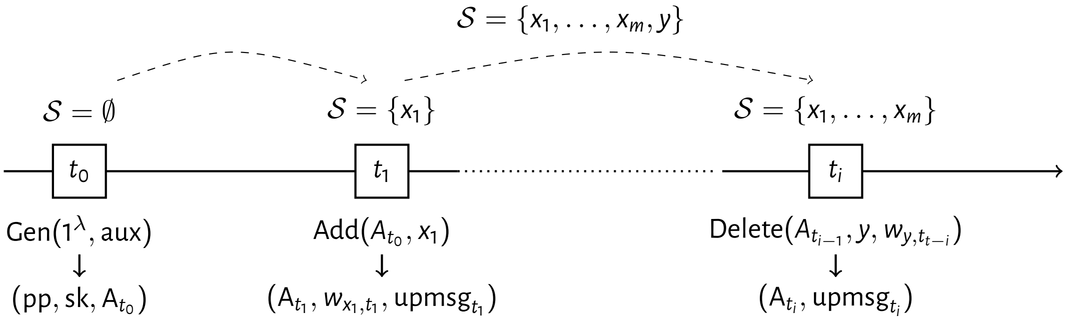

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- $\mathsf{NonMemWitCreate}(\mathsf{A}_t, x, \{\mathsf{upmsg}_i\}_{i=1}^{t}) \to \bar{w}_{x,t}$
- $\mathsf{NonMemWitUp}(x, \bar{w}_{x,t}, \mathsf{upmsg}_{t+1}) \to \bar{w}_{x,t+1}$
- $\mathsf{NonMemVerify}(\mathsf{A}_t, x, \bar{w}_{x,t}) \to \mathsf{Accept}/\mathsf{Reject}$

Dynamic Accumulator's Properties

Definition (Compactness)

$|\mathsf{A}| = \mathsf{poly}(\lambda), |w_{x,t}| = |\bar{w}_{x,t}| = \mathsf{poly}(\lambda, |x|)$

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Definition (Correctness–Informal)

An accumulator scheme is correct if given A_t, $(x, w_{x,t})$, and $(y, \bar{w}_{y,t})$ such that $w_{x,t}, \bar{w}_{y,t}$ are up-to-date:

- $(x, w_{x,t})$ pass MemVerify with overwhelming probability
- $(y, \bar{w}_{y,t})$ pass NonMemVerify with overwhelming probability

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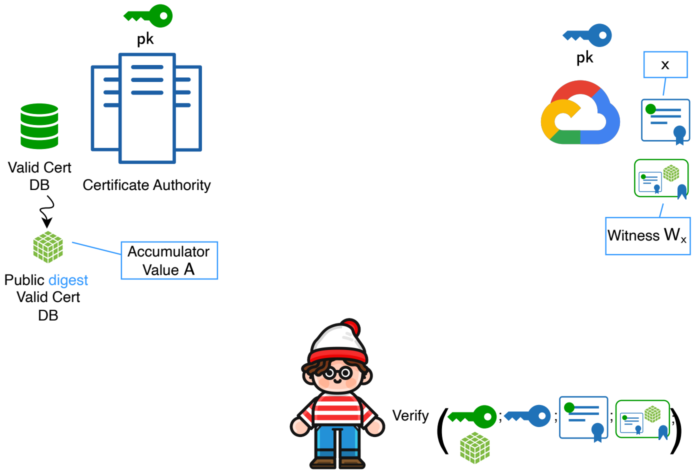
Definition (Security–Informal)

An accumulator scheme is secure if for all poly-time adversary $\mathcal{A}$:

- It is *hard* to output a valid w_x for any $x \notin \mathcal{S}$
- It is *hard* to output a valid $\bar{w}_y$ for any $y \in \mathcal{S}$

Dynamic Accumulator in WebPKI Certificate Revocation



Prior Works

- Benaloh and de Mare [BdM94]: a static positive accumulator for random integers based on the hardness of computing arbitrary roots in RSA groups.

$$A \leftarrow u^{\prod_{i=1}^{n} x_i} \bmod n \quad \text{and} \quad w_{x_i} = A^{1/x_i}$$

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Working over PRIMES requires hashing to prime integers in practice:

Try $r \in \{0, \ldots, N\}$ until $H(x; r)$ is prime. Then, accumulate $A' \leftarrow A^{H(x;r)}$.

Based on the Prime Number Theorem, this incurs an overhead of $O(\log N)$

Prior Works

- Other proposals based on billinear-pairing [Ngu05, CKS09, ATSM09] that works over integers.
  - Require public parameters whose size is linear in the number of elements to be accumulated.
  - Reducing the size of public parameters is possible at the expense of requiring a trapdoor for Add and Delete.
- Other proposals based on Merkle-tree [CW09, RY16] that works over integers.
  - Witness size is logarithmic in the number of elements accumulated.
  - Supporting non-membership and deletion is non-trivial.

Our Contributions

- RSA-based unviersal and positive dynamic accumulators defined over large odd integers.
- Security holds under the strong RSA assumption in the random oracle model.
- **At least** three times faster than RSA-based accumulators defined over primes for $\lambda = 128$.
- A variant of Wesolowski's Proof of Exponentation [Wes20], called *SimPoE*, that does not require hashing to primes.
- We showed how to aggregate (non-)membership witnesses and use *SimPoE* to reduce the verification time of aggregated witnesses.

Outline

1. Universal Accumulator: Gen, Add, Delete, MemVerify, MemWitUp
2. Wesolowski's Proof of Exponentation without hashing to primes, i.e., SimPoE

Some Number Theory notions

- $\mathsf{Odds}(2^{\ell-1}, 2^{\ell}-1) \stackrel{\text{def}}{=} \{2^{\ell-1} \leq n \leq 2^{\ell}-1 : n \bmod 2 = 1\}$.
- $P^{+}(a)$: return the largest prime factor of a.

Based on [dB51, HT93],

Lemma (Informal)

Given a sufficiently large $\ell \in \mathbb{N}$, *If* $a \leftarrow\$ \mathsf{Odds}(2^{\ell-1}, 2^{\ell}-1)$, *Then*

$$\Pr\left[P^{+}(a) > 2^{\sqrt[4]{\ell}}\right] \geq 1 - O\left(2^{-\sqrt[4]{\ell}}\right)$$

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$$\Pr\left[P^{+}(a) > 2^{\sqrt[4]{\ell}}\right] \geq 1 - O\left(2^{-\sqrt[4]{\ell}}\right)$$

In other words, If $a \leftarrow_{\$} \text{Odds}(2^{\ell-1}, 2^{\ell}-1)$, then with overwhelming probability, a has a large prime factor with $\Omega(\sqrt[4]{\ell})$-bits

Some Number Theory notions

Corollary (Informal)

For $m \in \mathbb{N}$, *given a sufficiently large* $\ell \in \mathbb{N}$, *and* $a_1, a_2, \ldots, a_m \sim U\left(\mathrm{Odds}(2^{\ell-1}, 2^{\ell}-1)\right)$. *Then,*

$$\Pr\left[P^{+}(a_i) \mid \prod_{j \in [m] \setminus \{i\}} a_j\right] \leq m^2 O\left(2^{-\sqrt[4]{\ell}}\right) \quad \forall i \in [m]$$

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In other words, If you select $a_1, \ldots, a_m \in \text{Odds}(2^{\ell-1}, 2^{\ell}-1)$ uniformly at random, then the probability that $a_i \mid \prod_{j=1, j \neq i}^{m} a_j$ is negligible

Our Universal Accumulator Construction

$$H : \{0,1\}^* \to \mathrm{Odds}(2^{\ell-1}, 2^{\ell} - 1)$$

$\ell = \mathrm{poly}(\lambda)$, s.t. for all $x \in \{0,1\}^*$, $\mathbf{P}^{+}(\mathbf{H}(\mathbf{x})) > \mathbf{2}^{\sqrt[4]{\ell}}$ with overwhelming probability.

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- $\mathsf{Gen}(1^\lambda, \perp)$:
  1. Return $\mathsf{pp} = (n, u)$, $\mathsf{sk} = (p-1)(q-1)$, and $\mathsf{A}_0 \leftarrow u \in \mathrm{QR}_n$.
- $\mathsf{Add}(\mathsf{pp}, \mathsf{A}, x)$:
  1. Parse pp as (n, u).
  2. Compute $\mathsf{A}' \leftarrow \mathsf{A}^{\mathbf{H}(\mathbf{x})} \bmod n$.
  3. Let $\mathbf{s} = (1)$, $w_x = (\mathsf{A}, \mathbf{s})$ and $\mathsf{upmsg} = (\mathsf{add}, H(x), \mathbf{1}, \mathsf{A}, \mathsf{A}')$.
  4. Return A', w_x, and upmsg.

Our Universal Accumulator Construction

- $\mathsf{Delete}(\mathsf{pp}, \mathsf{sk}, \mathsf{A}, x, w_x)$:
  1. Parse pp as (n, u)
  2. Compute $\gamma \leftarrow 1/H(x) \bmod \mathsf{sk}$, and let $\delta = 1$.
  3. Compute $\mathsf{A}' \leftarrow \mathsf{A}^{\gamma} \bmod n$.
  4. Let upmsg $= (\mathsf{del}, H(x), \delta, \mathsf{A}, \mathsf{A}')$.
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Note: We can use w_x to avoid using sk

How to update membership witnesses

We recall Camenisch-Lysyanskaya [CL02] membership update algorithm.

- $\mathsf{MemWitUp}(\mathsf{pp}, x, w_x, \mathsf{upmsg})$:
  1. Parse pp as (n, u), w_x as $(\mathtt{w}, \mathbf{s})$, and upmsg as $(\mathsf{op}, H(y), \delta, \mathsf{A}, \mathsf{A}')$.
  2. If $\mathsf{op} = \mathsf{add}$, *// After* Add, $\mathsf{A}' = \mathsf{A}^{H(y)}$
     - compute $\mathtt{w}' \leftarrow \mathtt{w}^{H(y)} \bmod n$, and let $w'_x = (\mathtt{w}', \mathbf{s})$.

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  3. Else if $\mathsf{op} = \mathsf{del}$, do: // *After* Delete, $\mathsf{A}' = \mathsf{A}^{1/H(y)}$
     1. Compute $a, b \in \mathbb{Z}$ such that $aH(x) + bH(y) = \gcd(H(x), H(y))$.
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Correctness issue

$$\underset{\sim\sim}{(\mathtt{w}')^{H(x)}} = ((\mathsf{A}')^a \mathtt{w}^b)^{H(x)} = ((\mathsf{A}')^a \mathtt{w}^b)^{H(x)H(y)(1/H(y))} = (\mathsf{A}^{1/H(y)})^{\gcd(H(x),H(y))} = \underset{\sim\sim}{(\mathsf{A}')^{\gcd(H(x),H(y))}}$$

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  3. Else if op $=$ del, do: *// After* Delete, $\mathsf{A}' = \mathsf{A}^{1/H(y)}$
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Fixing the issue

Observe that $(\mathtt{w}')^{\mathbf{H(x)}/\gcd(\mathbf{H(x)},\mathbf{H(y)})} = \mathsf{A}'$. What if we *consider* $\frac{H(x)}{\gcd(H(x),H(y))}$ as our accumulated element?

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Note that $P^+(H(x)) = P^+(\frac{H(x)}{\gcd(H(x),H(y))})$.

Our Universal Accumulator Construction

We have x with witness $w_x = (\mathtt{w}, \mathbf{s})$. *Assume* $\mathbf{s} = ()$

- y_1 was deleted
  Compute $a_1, b_1 \in \mathbb{Z}$ such that $a_1 H(x) + b_1 H(y_1) = \gcd(H(x), H(y_1))$
  Compute $\mathtt{w}' \leftarrow (\mathrm{A}')^{a_1} \mathtt{w}^{b_1} \bmod n$, and $\mathbf{s}' \leftarrow \mathbf{s} \| (\gcd(H(x), H(y_1)))$

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  To verify, parse $\mathbf{s}'$ as $\gcd(H(x), H(y_1))$, compute $\mathtt{x} \leftarrow \frac{H(x)}{\gcd(H(x),H(y_1))}$. Finally, check

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- y_2 was deleted
  Compute $a_2, b_2 \in \mathbb{Z}$ such that $a_2 \mathtt{x} + b_2 H(y_1) = \gcd(\mathtt{x}, H(y_2))$
  Compute $\mathtt{w}'' \leftarrow (\mathsf{A}'')^{a_2} \mathtt{w}^{b_2} \mod n$, and $\mathbf{s}'' \leftarrow \mathbf{s}' \| (\gcd(\mathtt{x}, H(y_2)))$

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  Compute $a_1, b_1 \in \mathbb{Z}$ such that $a_1 H(x) + b_1 H(y_1) = \gcd(H(x), H(y_1))$
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  To verify, parse $\mathbf{s}'$ as $\gcd(H(x), H(y_1))$, compute $\mathtt{x} \leftarrow \frac{H(x)}{\gcd(H(x),H(y_1))}$. Finally, check

$$(\mathtt{w}')^{\mathtt{x}} \stackrel{?}{=} \mathsf{A}'$$

- y_2 was deleted
  Compute $a_2, b_2 \in \mathbb{Z}$ such that $a_2 \mathtt{x} + b_2 H(y_1) = \gcd(\mathtt{x}, H(y_2))$
  Compute $\mathtt{w}'' \leftarrow (\mathsf{A}'')^{a_2} \mathtt{w}^{b_2} \mod n$, and $\mathbf{s}'' \leftarrow \mathbf{s}' \| (\gcd(\mathtt{x}, H(y_2)))$

// Note: $P^+(H(x)) = P^+(\mathtt{x})$

Our Universal Accumulator Construction

New Membership update algorithm:

- $\mathsf{MemWitUp}(\mathsf{pp}, x, w_x, \mathsf{upmsg})$:
  1. Parse pp as (n, u), w_x as $(\mathtt{w}, \mathbf{s})$, and upmsg as $(\mathsf{op}, H(y), \delta, \mathsf{A}, \mathsf{A}')$.
  2. If $\mathsf{op} = \mathsf{add}$, compute $\mathtt{w}' \leftarrow \mathtt{w}^{H(y)} \bmod n$, and let $w'_x = (\mathtt{w}', \mathbf{s})$.
  3. Else if $\mathsf{op} = \mathsf{del}$, do:
     1. Compute $\mathtt{x} \leftarrow H(x) / \prod_{i=1}^{|\mathbf{s}|} \mathbf{s}[i]$.
     2. Compute $a, b \in \mathbb{Z}$ such that $a\mathtt{x} + bH(y) = \gcd(\mathtt{x}, H(y))$.
     3. Compute $\mathtt{w}' \leftarrow (\mathsf{A}')^a \mathtt{w}^b \bmod n$.
     4. If $\gcd(\mathtt{x}, H(y)) \neq 1$, let $\mathbf{s}' \leftarrow \mathbf{s} \| (\gcd(\mathtt{x}, H(y)))$. Otherwise, let $\mathbf{s}' \leftarrow \mathbf{s}$.
     5. Let $w'_x = (\mathtt{w}', \mathbf{s}')$.
  4. Return w'_x.

Our Universal Accumulator Construction

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$\mathbf{s}$ is a tuple that contains **small/smooth** factors of $H(x)$ and $|\mathbf{s}| \leq \ell$, where ℓ is the output length of H.

Our Universal Accumulator Construction

//$P^+(H(x)) > 2^{\sqrt[4]{\ell}}$ with overwhelming probability.

- $\mathsf{MemVerify}(\mathsf{pp}, \mathsf{A}, x, w_x)$:
  1. Parse pp as (n, u), and w_x as $(\mathtt{w}, \mathbf{s})$.
  2. For $i \in [|\mathbf{s}|]$, if $\mathbf{s}[i] > 2^{\sqrt[4]{\ell}}$, return 0.
  3. Compute $\mathtt{x} \leftarrow H(x) / \prod_{i=1}^{|\mathbf{s}|} \mathbf{s}[i]$.
  4. If $\mathtt{w}^{\mathtt{x}} \equiv \mathsf{A} \mod n$ return 1. Otherwise, return 0.

Line 2 ensures that $P^+(H(x)) = P^+(\mathtt{x})$.

Our Universal Accumulator Construction–Analysis

Definition (Correctness–Informal)

An accumulator scheme is correct if given A_t, $(x, w_{x,t})$, and $(y, \bar{w}_{y,t})$ such that $w_{x,t}, \bar{w}_{y,t}$ are up-to-date:

- $(x, w_{x,t})$ pass MemVerify with overwhelming probability
- $(y, \bar{w}_{y,t})$ pass NonMemVerify with overwhelming probability

Definition (Security–Informal)

An accumulator scheme is secure if for all poly-time adversary $\mathcal{A}$:

- It is *hard* to output a valid w_x for any $x \notin \mathcal{S}$
- It is *hard* to output a valid $\bar{w}_y$ for any $y \in \mathcal{S}$

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Our Universal Accumulator Construction–Analysis

Security

Definition (Strong RSA Assumption)

Given (u, n), with $u \in \mathbb{Z}_n^*$, output $(v, e) \in \mathbb{Z}_n^* \times \mathbb{Z}$ such that

$$v^e \equiv u \bmod n \quad \wedge \quad e > 1$$

Our Universal Accumulator Construction–Analysis

Security

Suppose adversary $\mathcal{A}$ outputs $\{x_1, \ldots, x_m\}$, $\mathsf{A} = u^{\prod_{i=1}^{m} H(x_i)}$, (x^*, w_{x^*}) such that $x^* \notin \{x_1, \ldots, x_m\}$.

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1. $H(x^*) \nmid \prod_{i=1}^{m} H(x_i)$ with overwhelming probability. //Corollary from $P^+(H(x)) > 2^{\sqrt[4]{\ell}}$ w.o.p
2. $w_{x^*} = (\mathtt{w}, \mathbf{s})$ is valid. Therefore, all components of $\mathbf{s}$ are $2^{\sqrt[4]{\ell}}$-smooth.

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$$\mathtt{w}^{\mathtt{x}^*} = \mathrm{A} = u^{\theta} \qquad (\clubsuit)$$

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$$(u^a \mathtt{w}^b)^{\tilde{\mathtt{x}}} = u^{a\tilde{\mathtt{x}}} u^{b\tilde{\theta}} = u$$

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$$(u^a \mathtt{w}^b)^{\tilde{\mathtt{x}}} = u^{a\tilde{\mathtt{x}}} u^{b\tilde{\theta}} = u$$

Therefore $u^a \mathtt{w}^b$ is an $\tilde{\mathtt{x}}$-root of u

Some Experimental results

λ	H_{Prime} length (bit)	H_{Prime} time (ms)	H_{Odd} length (bit)	H_{Odd} time (ms)
112	232	10.65	1440	0.48
128	264	13.62	1704	0.60
192	393	31.9	2896	1.07
256	521	52.31	4208	1.56

Table: H_{Prime} versus H_{Odd}.

Some Experimental results

λ	$\mathsf{Add}^{(H_{\mathsf{Prime}},\mathsf{sk})}$ time (ms)	$\mathsf{Add}^{(H_{\mathsf{Odd}},\mathsf{sk})}$ time (ms)	$\mathsf{Add}^{H_{\mathsf{Prime}}}$ time (ms)	$\mathsf{Add}^{H_{\mathsf{Odd}}}$ time (ms)
112	12.83	2.73	11.06	1.96
128	20.37	7.38	14.27	3.98
192	99.48	68.84	35.34	25.10
256	456.10	402.71	65.97	110.6

Table: Comparison of different Add algorithms. $\mathsf{Add}^{(H,\mathsf{sk})}$ represents the addition procedure that uses the secret key sk and H as the underlying hash function, and Add^{H} represents the addition procedure that is performed without sk using H as the underlying hash function.

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Outline

1. **Universal Accumulator:** Gen, Add, Delete, MemVerify, MemWitUp ✓
2. Wesolowski's Proof of Exponentation without hashing to primes, i.e., SimPoE

Proof of Exponentation

Definition

A Proof of Exponentation (PoE) is an interactive protocol (argument) for the language

$$\mathcal{L}_{\mathsf{PoE},\mathbb{G}} = \{(v, u, e) \in \mathbb{G}^2 \times \mathbb{Z} : v^e = u\}$$

where $\mathbb{G}$ is a group of unknown order.

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where $\mathbb{G}$ is a group of unknown order.

We recall Wesolowski's PoE [Wes20]:

Initialization:

1. Sample and output a group $\mathbb{G}$ of unknown order.
2. **Statement:** $(v, u, e) \in \mathbb{G}^2 \times \mathbb{Z}$.

Interaction:

1. V samples $c \leftarrow\$ \ \mathtt{PRIMES}(2^\lambda)$ and sends it to P.
2. P computes $\pi \leftarrow v^{\lfloor e/c \rfloor}$ and sends it to V.
3. V computes $r \leftarrow e \mod c$. Then, it outputs 1 if $\pi^c v^r = u$. Otherwise, it outputs 0.

Proof of Exponentation : SimPoE

Definition

A Proof of Exponentation (PoE) is an interactive protocol (argument) for the language

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We present *SimPoE*:

Initialization:

1. Sample and output a group $\mathbb{G}$ of unknown order.
2. **Statement:** $(v, u, e) \in \mathbb{G}^2 \times \mathbb{Z}$**.**

Interaction:

1. V samples $c \leftarrow\$ \mathsf{Odds}(2^{\ell-1}, 2^{\ell} - 1)$ and sends it to P.
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3. V computes $r \leftarrow e \mod c$. Then, it outputs 1 if $\pi^c v^r = u$. Otherwise, it outputs 0.

Thank You

https://ia.cr/2024/505