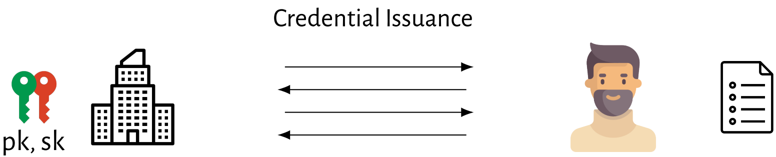
# Lattice-Based Accumulator and Application to Anonymous Credential Revocation

Victor Youdom Kemmoe    Anna Lysyanskaya    Ngoc Khanh Nguyen

# Motivation

## Anonymous Credentials [CL02 a; BBC+24 ]



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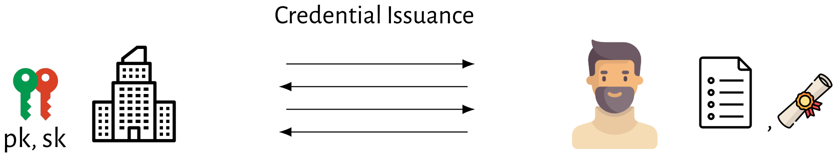

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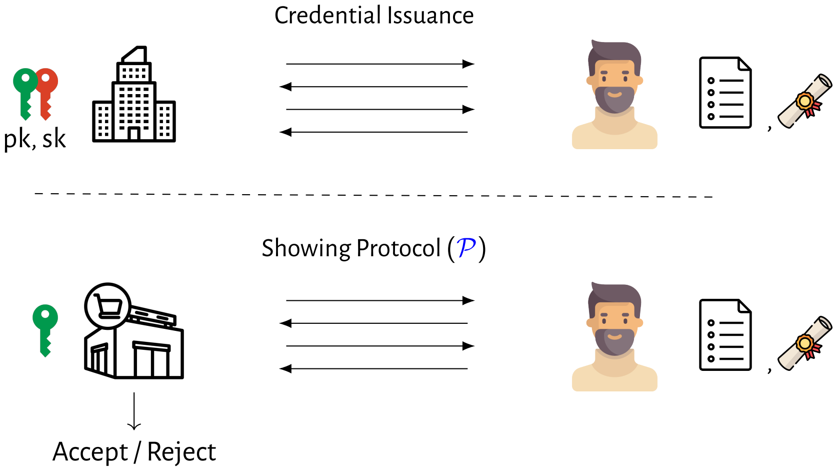

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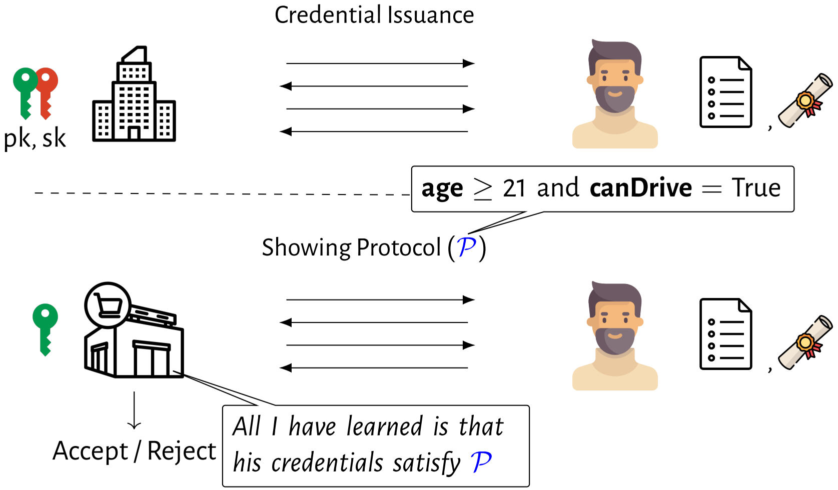

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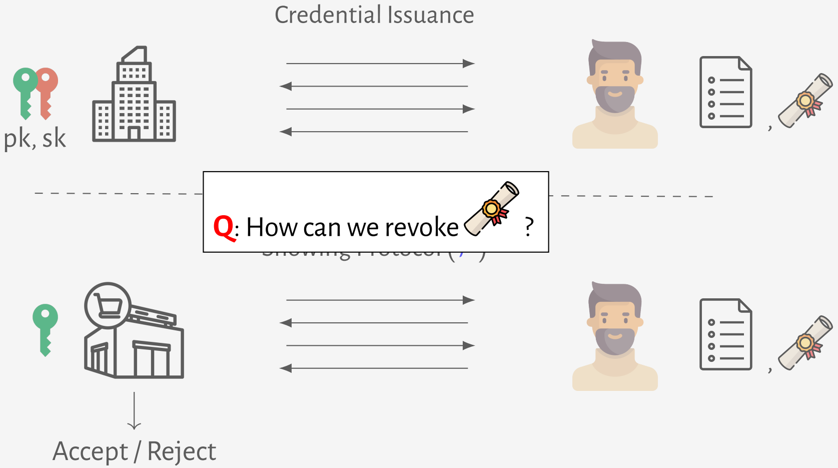

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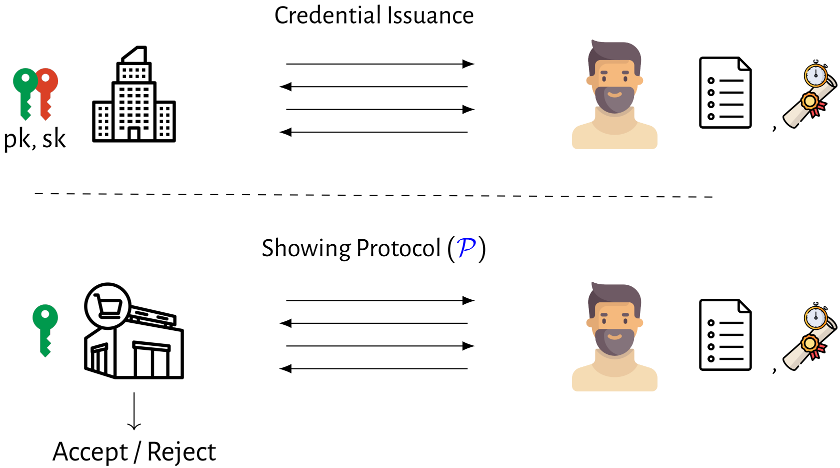

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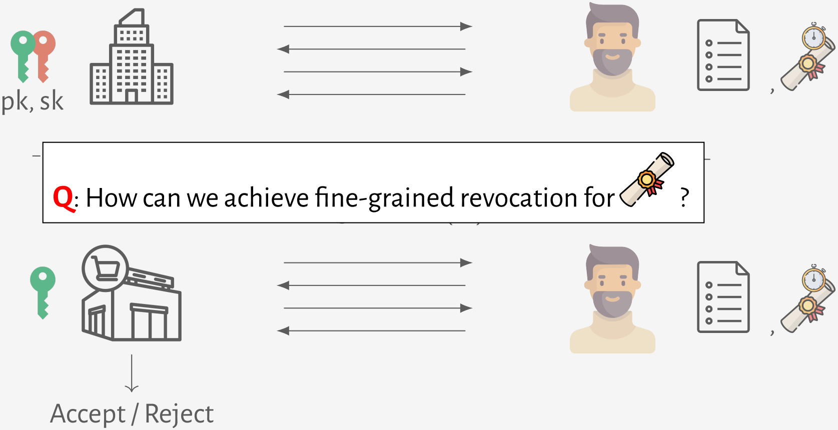
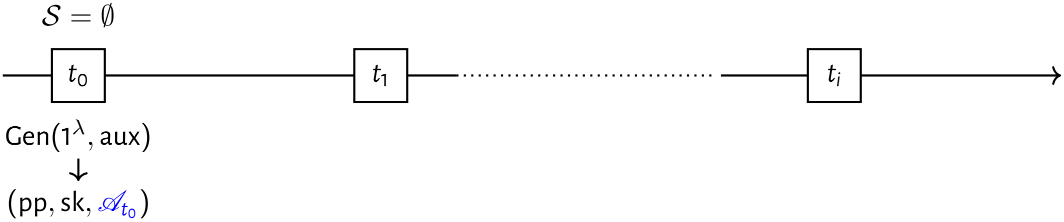



**Q**: How can we achieve fine-grained revocation for ?

# Positive Dynamic Accumulator

## Syntax [BCD+17 ; DHS15 ; KL24 ]



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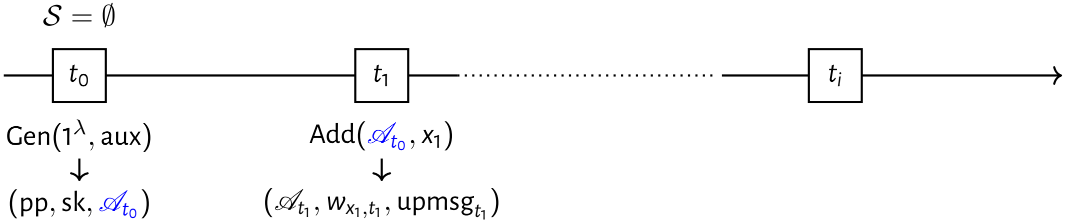

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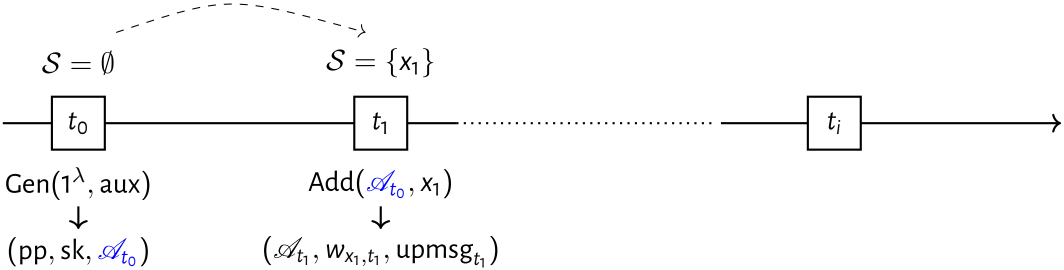

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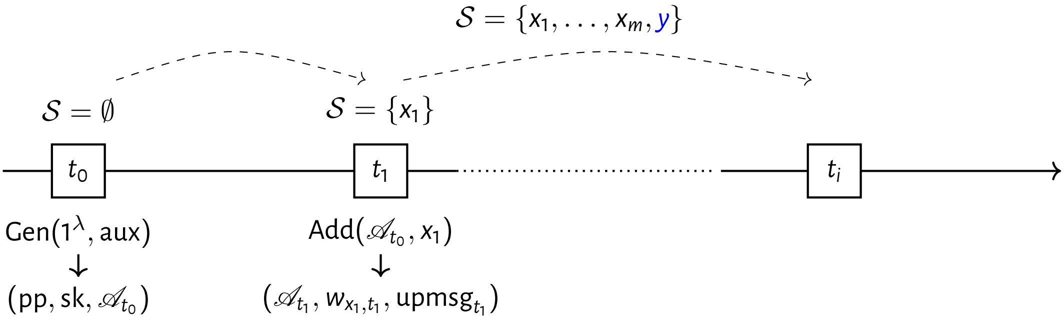

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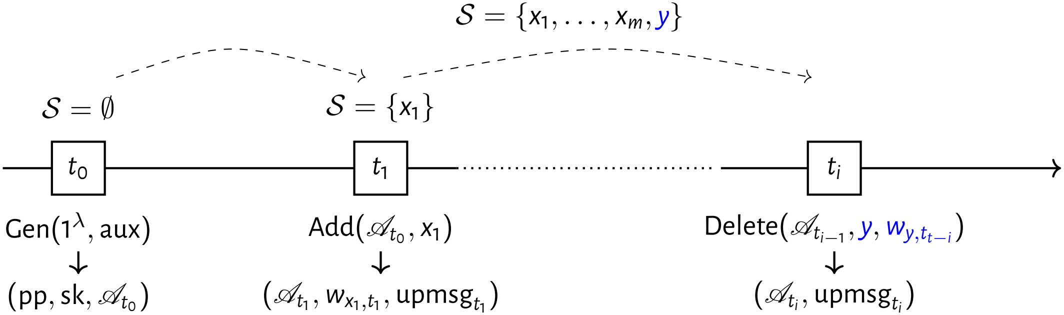

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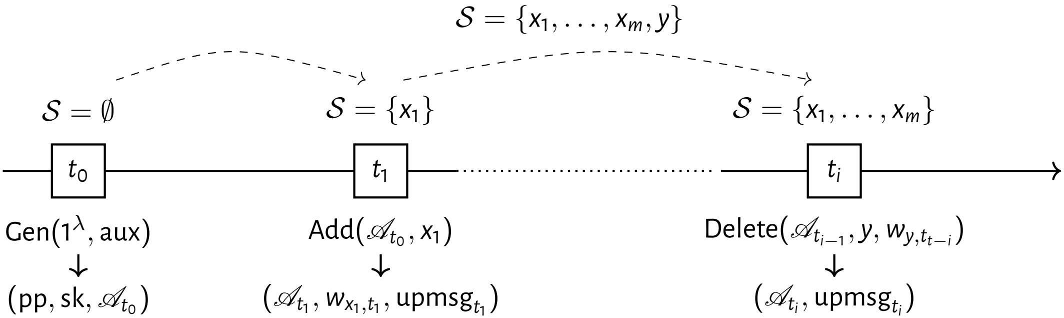

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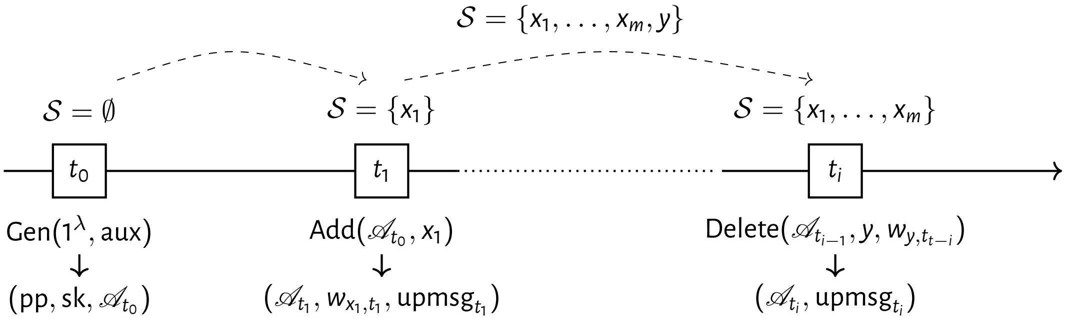


- $\mathsf{MemWitUp}(x, w_{x,t}, \mathsf{upmsg}_{t+1}) \rightarrow w_{x,t+1}$
- $\mathsf{MemVerify}(\mathscr{A}_t, x, w_{x,t}) \rightarrow \mathsf{Accept}/\mathsf{Reject}$

# Positive Dynamic Accumulator

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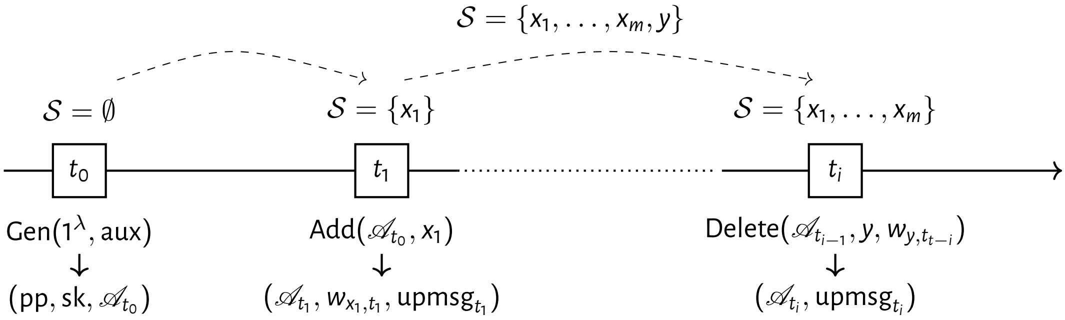
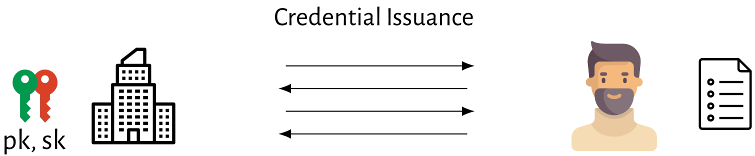



- $\mathsf{MemWitUp}(x, w_{x,t}, \mathsf{upmsg}_{t+1}$
- $\mathsf{MemVerify}(\mathscr{A}_t, x, w_{x,t}) \to$ Ac

- **Compactness**: $|\mathscr{A}| = \mathsf{poly}(\lambda)$, $|w_{x,t}| = \mathsf{poly}(\lambda, |x|)$
- **Security**: *Hard* to produce a $w_x$ for $x \notin \mathcal{S}$
- **Communication efficiency**: $|\mathsf{upmsg}| = O(\#\mathsf{Del})$

# Positive Dynamic Accumulator in ACs revocation



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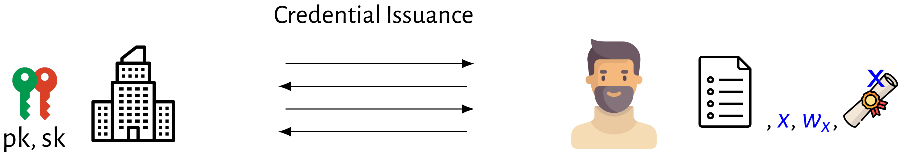

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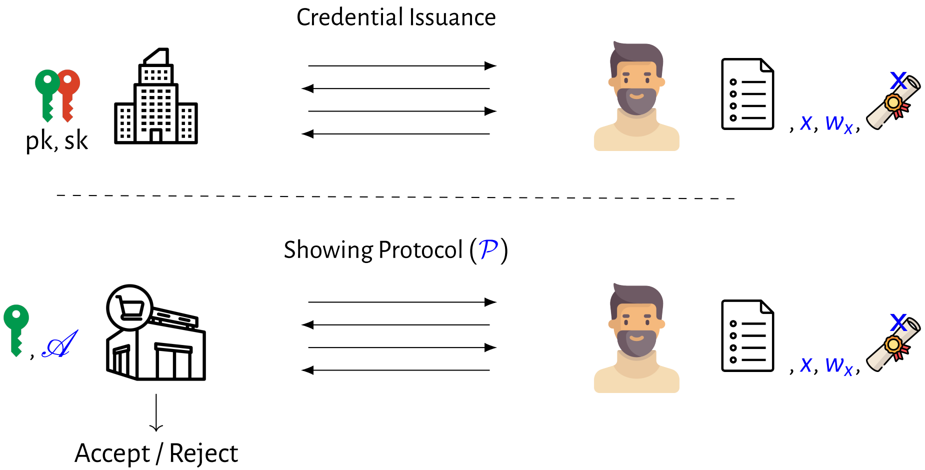

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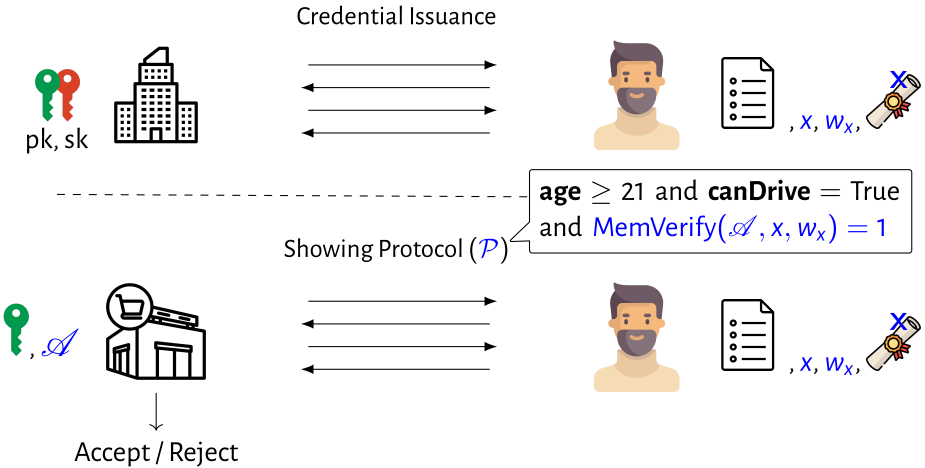

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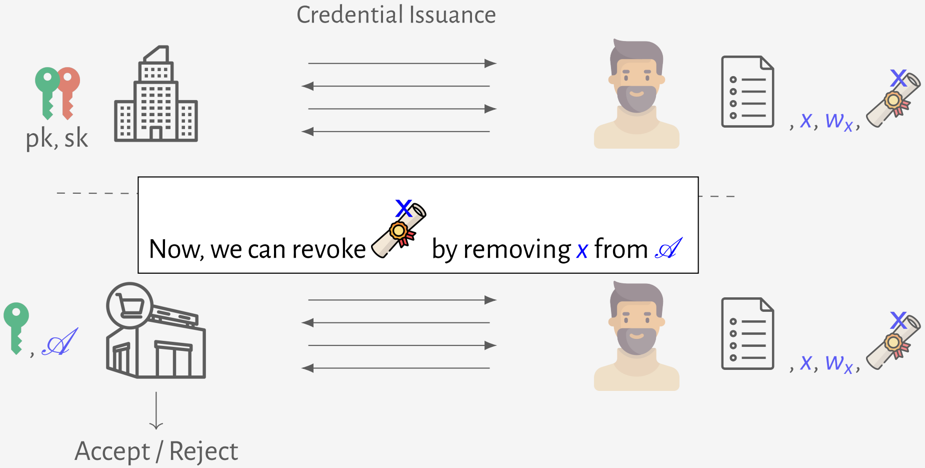

# Prior works on Positive Dynamic Accumulators

| Scheme | Assumption | $\|w\|$ | $\|\mathsf{upmsg}\|_{\mathsf{Add}}$ | $\|\mathsf{upmsg}\|_{\mathsf{Del}}$ | $\|\mathsf{pp}\|$ |
|---|---|---|---|---|---|
| [CL02 b; LLX07 ; KL24 ] | Strong RSA | $\ell \cdot \mathsf{poly}(\lambda)$ | $\ell^*$ | $\ell$ | $\mathsf{poly}(\lambda)$ |
| [BCD+17 ; KL24 ] | Strong RSA | $\ell \cdot \mathsf{poly}(\lambda)$ | — | $\ell$ | $\mathsf{poly}(\lambda)$ |
| [Ngu05 ; ATS+09 ; CKS09 ] | $q$-Strong DH | $\mathsf{poly}(\lambda)$ | $\ell^*$ | $\ell$ | $s \cdot \mathsf{poly}(\lambda)$ |
| [KB21 ; JML24 ] | $q$-Strong DH | $\mathsf{poly}(\lambda)$ | — | $\ell$ | $\mathsf{poly}(\lambda)$ |
| [PST+13 ; YAY+18 ; LLN+23 ] | M-SIS | $\mathsf{poly}(\lambda) \cdot \log s$ | $\mathsf{poly}(\lambda) \cdot \log s^*$ | $\mathsf{poly}(\lambda) \cdot \log s$ | $\mathsf{poly}(\lambda)$ |
| [ZYH24 ] | M-SIS | $\mathsf{poly}(\lambda)$ | $\mathsf{poly}(\lambda)^*$ | $\mathsf{poly}(\lambda)$ | $\mathsf{poly}(\lambda) \cdot s \log s$ |
| [CP23 ] | M-SIS | $\ell \cdot \mathsf{poly}(\lambda)$ | $\ell^*$ | $\ell$ | $\ell \cdot \mathsf{poly}(\lambda)$ |
| [CP23 ]+ [WW23 ] | $\ell$-Succinct M-SIS | $\mathsf{poly}(\lambda)$ | $\ell^*$ | $\ell$ | $\ell^2 \cdot \mathsf{poly}(\lambda)$ |
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- $\ell$: Input's bit length
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# Digital Signature

Let $\Sigma = (\mathsf{Gen}, \mathsf{Sign}, \mathsf{Verify})$ be a digital signature

- $\mathsf{Gen}(1^\lambda) \to (\mathsf{pk}, \mathsf{sk})$
- $\mathsf{Sign}(\mathsf{sk}, m) \to \sigma$
- $\mathsf{Verify}(\mathsf{pk}, m, \sigma) \to 1/0$

**Security**

It should be hard for an adversary to generate $(m^*, \sigma^*)$ given pk and $\{(m_i, \sigma_i)\}$ where $m^* \neq m_i$ for all $i$.

# Positive Dynamic Accumulator from Digital Signature

Let $\Sigma = (\mathsf{Gen}, \mathsf{Sign}, \mathsf{Verify})$ be a digital signature. In addition, suppose $\Sigma$ supports the following operations:

- $\mathsf{UpdatePK}(\mathsf{pk}, \mathsf{sk}, \bar{m}) \rightarrow (\mathsf{pk}', \mathsf{upmsg})$
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**Desiderata**

- $\text{Verify}(\text{pk}', m, \sigma'_m) = 1$ with overwhelming probability for any $m \neq \bar{m}$
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UpdatePK allows to *revoke* signatures on messages.

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- $\mathsf{Delete}(\mathsf{pk}, \mathsf{sk}, y)$:
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- $\mathsf{MemWitUp}(x, w_x, \mathsf{upmsg})$:
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This construction is communication efficient, i.e., $|\mathsf{upmsg}| = O(\#\mathsf{Del})$.

# Gadget Matrix

[MP12]

Let $R_q \supseteq \mathbb{Z}_q$ be a ring such that $R_q^m$ admits an $\ell_\infty$-norm

$$\mathbf{G} = \begin{bmatrix} 1, 2, 4, \ldots, 2^{k-1} & & & \\ & 1, 2, 4, \ldots, 2^{k-1} & & \\ & & \ddots & \\ & & & 1, 2, 4, \ldots, 2^{k-1} \end{bmatrix} \in R_q^{n \times nk}$$

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- $k = \lceil \log q \rceil$.
- There exists a *decomposition* function $\mathbf{G}^{-1} : R_q^n \to R_q^{nk}$ such that for any $\mathbf{u} \in R_q^n$, we have $\mathbf{G} \cdot \mathbf{G}^{-1}(\mathbf{u}) = \mathbf{u}$ and $\|\mathbf{G}^{-1}(\mathbf{u})\|_\infty = 1$

# Homomorphic Operations on Matrices

**[GSW13 ; BGG+14 ; CP23 ]**

For any $\ell \in \mathbb{N}$, let $\mathcal{F} = \{f_i : \{0,1\}^\ell \to \{0,1\}\}_{i \in \mathbb{N}}$ be a family of Boolean circuits. Then, there exist efficient algorithm EvalF and EvalFX such that for any $\mathbf{B} \in R_q^{n \times \ell m}$, $f \in \mathcal{F}$, and $x \in \{0,1\}^\ell$:

- $\mathsf{EvalF}(f, \mathbf{B}) \to \mathbf{B}_f$
- $\mathsf{EvalFX}(f, \mathbf{B}, x) \to \mathbf{H}_{f,x}$ with $\|\mathbf{H}_{f,x}\|_\infty = 1$

$$\text{s.t.} \quad (\mathbf{B} - x \otimes \mathbf{G}) \cdot \mathbf{H}_{f,x} = \mathbf{B}_f - f(x) \cdot \mathbf{G}$$

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$$\mathcal{F}_{\text{Indicator}} : \{\mathbb{1}_y : \{0,1\}^\ell \to \{0,1\}\}, \text{ where } \mathbb{1}_y(x) = \begin{cases} 1 \text{ if } x = y \\ 0 \text{ otherwise} \end{cases}$$

# Our Construction

## Communication efficient accumulator

$$\mathsf{pp} = (\mathbf{A} \in R_q^{n \times \bar{m}}, \mathbf{B} \in R_q^{n \times \ell m}), \mathsf{sk} = \mathbf{T}_{\mathbf{A}}, \mathscr{A}_0 \leftarrow\!\!\$\; R_q^{n \times m}$$

sk *allows to compute a low-norm matrix* $\mathbf{V} \leftarrow \mathsf{SamplePre}_{\mathsf{sk}}([\mathbf{A} \mid \bar{\mathbf{B}}], \mathbf{U})$ *s.t.* $[\mathbf{A} \mid \bar{\mathbf{B}}] \cdot \mathbf{V} = \mathbf{U}$ *for any* $\bar{\mathbf{B}}$.

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*Agrawal-Boneh-Boyen [ABB10 ] signature*

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  3. Return $w_x'$

- $\mathsf{MemVerify}(\mathsf{pp}, \mathscr{A}, x, w_x)$:
  1. Check if $[\mathbf{A} \mid \mathbf{B} - x \otimes \mathbf{G}] \cdot w_x = \mathscr{A}$ and $\|w_x\|_\infty$ is small

# Our Construction

## Communication efficient accumulator – Correctness

Let $x \in \{0,1\}^{\ell}$ with an updated witness $w'_x$ that was generated after deleting $y \neq x \in \{0,1\}^{\ell}$.
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- $\|w'_x\|_\infty = \|w_x\|_\infty + \left\|\mathbf{H}_{\mathbb{1}_y,\mathbf{B},x}\right\|_\infty = \|w_x\|_\infty + 1$

By setting the *noise* budget accordingly, we can support poly deletions.

# Our Construction

## Communication efficient accumulator – Instantiation

| **Scheme** | $q$ | #Add | #Del | $\lvert w_{\mathbf{x}} \rvert$ | $\lvert \text{upmsg} \rvert_{\text{Add}}$ | $\lvert \text{upmsg} \rvert_{\text{Del}}$ | $\lvert \mathscr{A} \rvert$ | $\lvert \text{pp} \rvert$ |
|---|---|---|---|---|---|---|---|---|
| [CP23 ] (M-SIS) | $\approx 2^{90}$ | $2^{32}$ | $2^{32}$ | 12MB | 4 B | 4 B | 45KB | 14.2MB |
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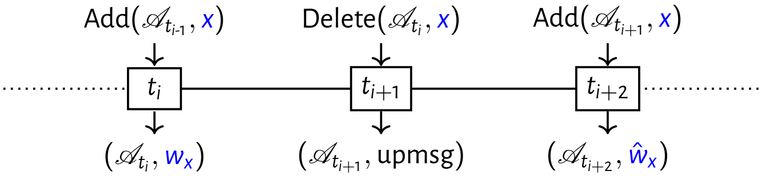

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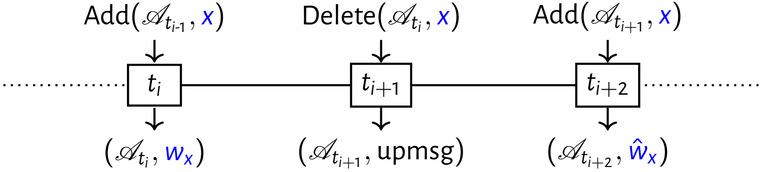


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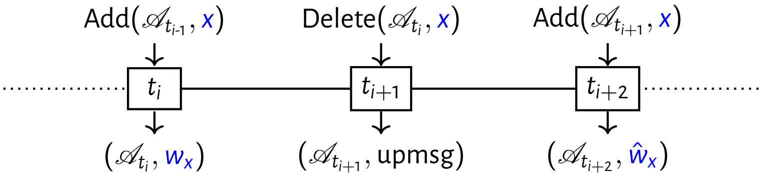


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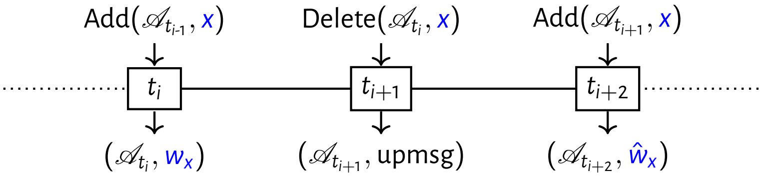


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**Theorem**

*If the replacement-free condition holds and the (module) Short Integer Solution problem is hard, then our construction is a selectively secure communication efficient positive dynamic* accumulator.*

# Security Analysis

**Short Integer Solution $(n, m, \beta)$**

Given $\bar{\mathbf{A}} \leftarrow\!\!\$\ R_q^{n\times m}$, find $\mathbf{v} \neq \mathbf{0}$ such that $\|\mathbf{v}\| \leq \beta$ and

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**Note:** Under the replacement-free condition, these two cases are sufficient.

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**Note**: A replacement-free selectively secure accumulator is sufficient for Anonymous Credential Revocation.

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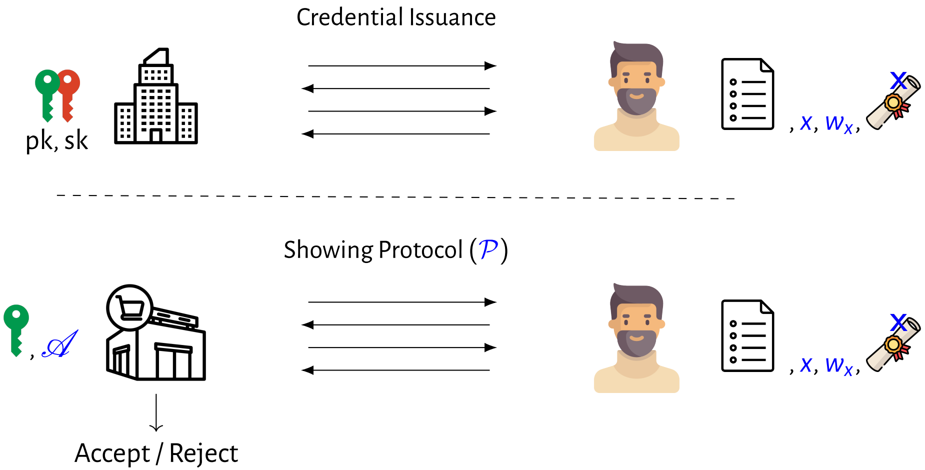

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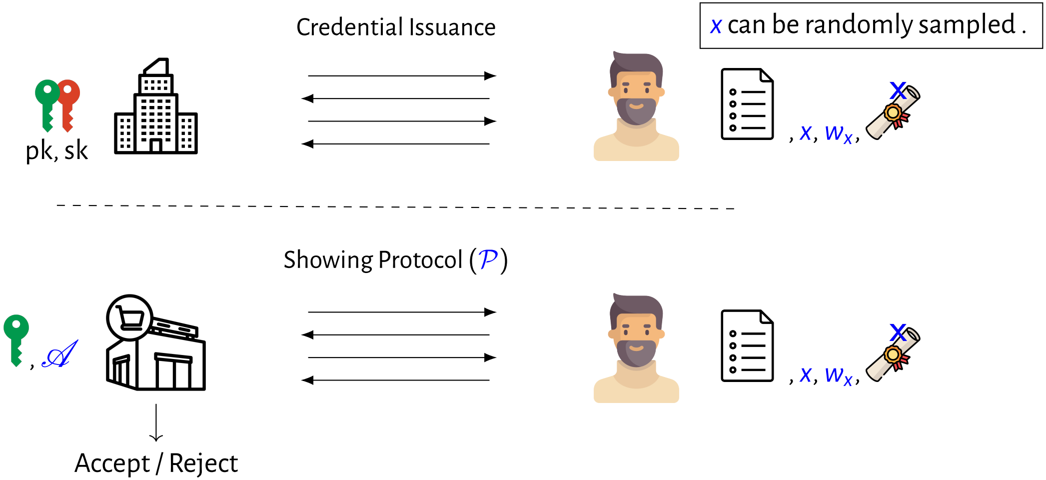

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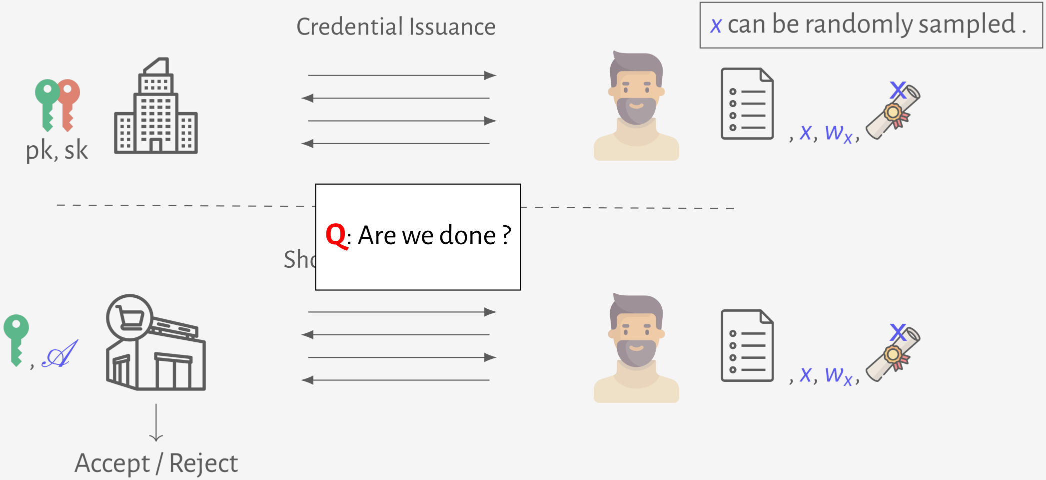

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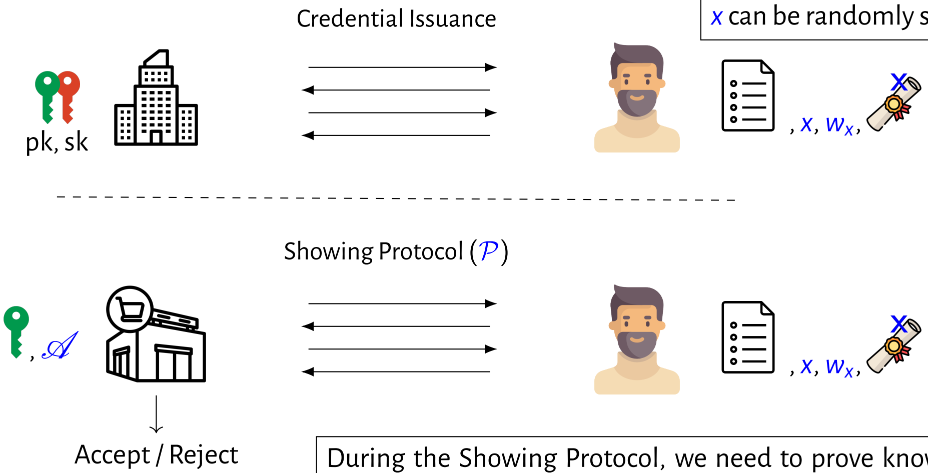
$x$ can be randomly sampled .




During the Showing Protocol, we need to prove knowledge of $x$ and $w_x$ s.t. MemVerify($\mathscr{A}$, $x$, $w_x$) $= 1$.

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From Lattice-based zero-knowledge proofs [Lyu12 ; ENS20 ; LNP+21 ; LNP22 ; BS23 ], we know how to prove knowledge of **v** such that

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How can we handle $x$?

- Compute a commitment $\mathrm{Com}(x; r)$ and produce a proof $\pi_{\mathrm{Com}} = (\mathbf{w}, c, z)$.
- From $z$, we can extract $z_x = y_x + c \cdot x$ such that

$$[c\mathbf{A} \mid c\mathbf{B} - z_x \otimes \mathbf{G}] \cdot w_x = c\underbrace{[\mathbf{A} \mid \mathbf{B} - x \otimes \mathbf{G}] \cdot w_x}_{\mathscr{A}} + [\mathbf{0} \mid -y_x \otimes \mathbf{G}] \cdot w_x$$

# Thank You!

https://ia.cr/2025/1099

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